Column Generation for WDM Optical Network Design

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Outline

- Basic concepts
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- Background
- Branch-And-Price (BP) Algorithm
  - Column Generation (CG)
  - Branching Strategy
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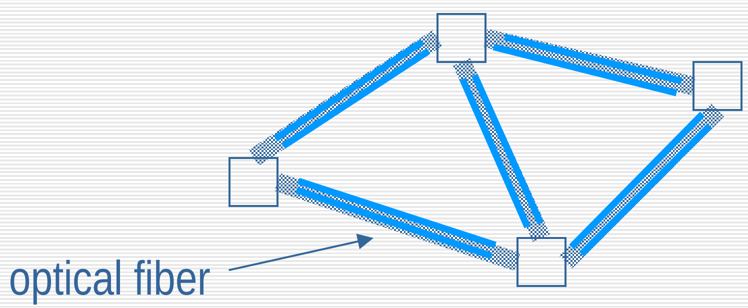
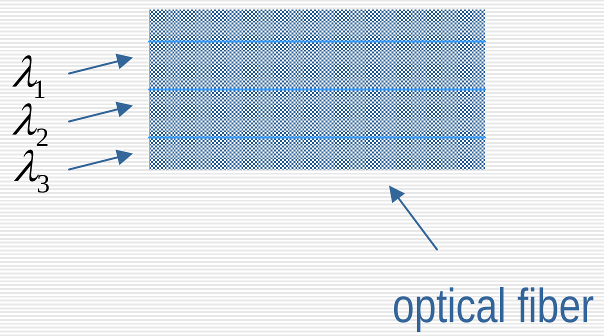
Basic Concepts

in WDM optical network design

- Optical fibers interconnect nodes in the network



- WDM – multiple signals carried over the same fiber at different frequencies (wavelengths)

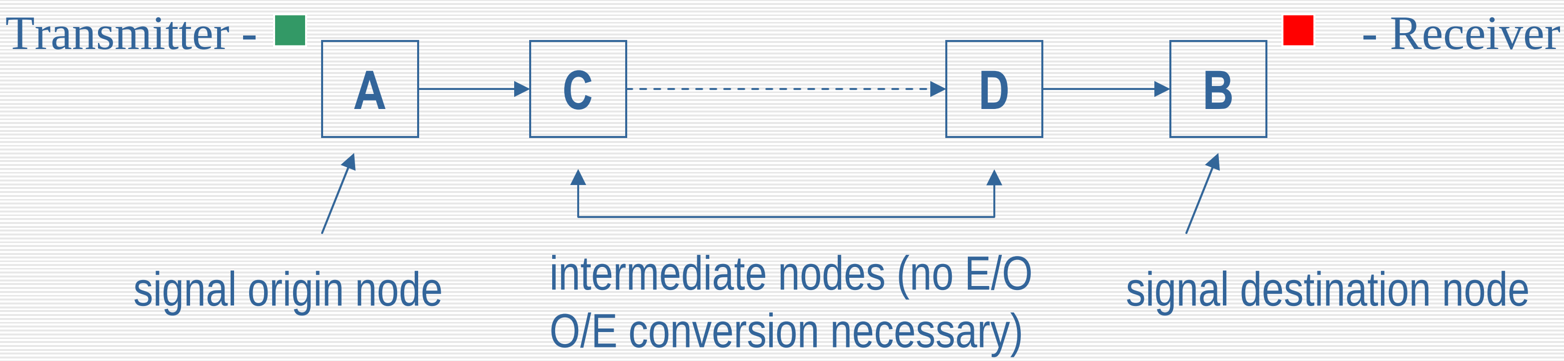


Basic Concepts

in WDM optical network design

Node Equipment

- Single signal example

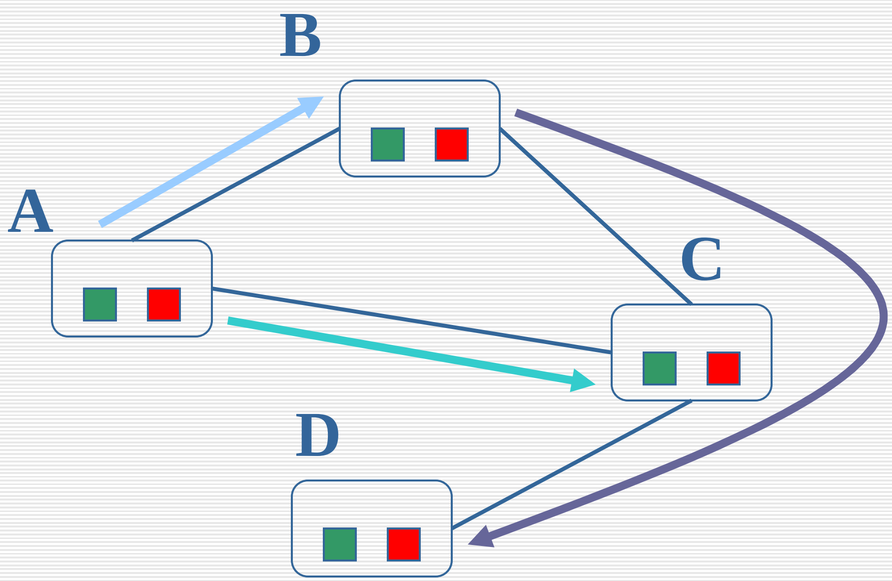


- Assumption: All nodes are equipped with wavelength converters ⇒ we do not have to worry about wavelength assignment (so, signal A → B could be sent on different wavelengths on each of the segments A → C, C → D, D → B)

Basic Concepts

in WDM optical network design

Notion of lightpaths and logical topology



■ - Transmitter

■ - Receiver

— - Fiber with capacity of 1 TU

<u>Traffic requests</u>:

A → B 0.3 TU's

A → C 0.9 TU's

A → D 0.2 TU's

- Def: **Lightpath** (lp) is a path in the physical topology used to carry traffic requests. It requires a transmitter at the path origin, and a receiver at the path destination (lps in the example: A → B, A → C, B → D)
- Def: **Logical Topology** is a collection of all lps established in the physical layer of the optical network.

Problem Definition

- Given physical topology of the WDM optical network:
  - Number and capacity of fibers
  - Capacity of lightpaths that can be created on the fibers
  - Number of transmitters and receivers at each node
  - Traffic matrix (demand between all pairs of nodes)

- Determine **logical topology** (routing of lightpaths over the physical topology) and **routing of traffic flow** over the logical topology so that network performance is optimized.

Additional Assumptions

- No flow bifurcation for a given traffic request
- Wavelength conversion is possible (at no cost) at all nodes in the network
- Performance measures considered:
  - Lost traffic
  - Quantity / cost of node equipment
  - Average hop distance over all flow paths in the network

Background

- Exact MIP formulations for WDM OND problem are too difficult to solve
  - Banerjee and Mukherjee (2000) - WDM OND with wavelength conversion (problems solved include networks with up to 20 nodes, at most 1 fiber between pairs of nodes, and pre-specified set of lightpaths)
  - Krishnaswamy and Sivarajan (2001) – WDM OND without wavelength conversion (problems solved include networks with up to 6 nodes)
- Large number of heuristic algorithms – an extensive survey – Dutta and Rouskas (2000)

Background

- The WDM OND problem with wavelength conversion can be seen as a 2-layer ODI MCF problem with node degree constraints – a generalization of a standard ODI MCF problem
- ODI MCF problem can be efficiently solved in networks of moderate size using branch and price and cut algorithm – Barnhart et al. (2000)

Path-based formulation for the WDM OND problem

- PB-MIP1

$$Min \quad \sum_{\forall (s,d)} T^{(s,d)} H^{(s,d)}$$

Subject to:

$$\sum_{z:O(z)=i} X_z \le \Delta_t^i \qquad \forall i \in V$$

$$\sum_{z:D(z)=j} X_z \le \Delta_r^j \qquad \forall j \in V$$

$$\sum_{z:(i,j);l \in z} X_z \le L \qquad \forall (i,j);l$$

$$X_z - \sum_{p:z \in p} T^{(s,d)} f_p^{(s,d)} \ge 0 \qquad \forall z$$

$$\sum_p f_p^{(s,d)} + H^{(s,d)} = 1 \qquad \forall (s,d)$$

$$f_p^{(s,d)} \in B^1 \qquad \forall p,(s,d)$$

$$X_z \in B^1 \qquad \forall z$$

Additional constraints

$$X_z - \sum_{p:z \in p} f_p^{(s,d)} \ge 0 \qquad \forall z,(s,d)$$

$$X_z \in R_+^1 \qquad \forall z$$

Path-based formulation for the WDM OND problem

- PB-MIP2

$$Min \quad \sum_{\forall (s,d)} T^{(s,d)} H^{(s,d)}$$

Subject to: *dual v.*

$$\sum_{\forall (s,d), p:\exists z \in p: O(z)=i} T^{(s,d)} f_p^{(s,d)} + \sum_{z:O(z)=i} Y_z \le \Delta_t^i \quad a_i \quad \forall i \in V$$

$$\sum_{\forall (s,d), p:\exists z \in p: D(z)=j} T^{(s,d)} f_p^{(s,d)} + \sum_{z:D(z)=j} Y_z \le \Delta_r^j \quad b_j \quad \forall j \in V$$

$$\sum_{\forall (s,d), p:\forall z \in p:(i,j);l \in z} T^{(s,d)} f_p^{(s,d)} + \sum_{z:(i,j);l \in z} Y_z \le L \quad d_{(i,j);l} \quad \forall (i,j);l$$

$$\sum_{\forall (s,d), p:\forall z \in p} T^{(s,d)} f_p^{(s,d)} + Y_z + G_z = 1 \quad g_z \quad \forall z$$

$$X_z + G_z = 1 \quad \forall z$$

$$X_z - \sum_{p:z \in p} T^{(s,d)} f_p^{(s,d)} \ge 0 \quad r_z^{(s,d)} \quad \forall z$$

$$\sum_p f_p^{(s,d)} + H^{(s,d)} = 1 \quad w^{(s,d)} \quad \forall (s,d)$$

$$f_p^{(s,d)} \in B^1 \quad \forall p,(s,d)$$

$$X_z \in Z_+^1 \quad \forall z$$

Additional constraints

$$X_z - \sum_{p:z \in p} f_p^{(s,d)} \ge 0 \quad v_z \quad \forall z,(s,d)$$

$$X_z \in R_+^1 \quad \forall z$$

Column Generation for PB-MIP2

- Reduced cost for any flow path variable is:

$$\sum_{\forall z \in p} \left(-T^{(s,d)} a_i - T^{(s,d)} b_j - \sum_{\forall (i,j);l \in z} T^{(s,d)} d_{(i,j);l} - T^{(s,d)} g_z + r_z^{(s,d)} + T^{(s,d)} v_z\right) - w^{(s,d)}$$

- To identify potential new flow paths we can solve the following problem for each commodity:

$$(*) \quad \underset{\forall z}{Min} \sum_{\forall z \in p} \left(-T^{(s,d)} a_i - T^{(s,d)} b_j - \sum_{\forall (i,j);l \in z} T^{(s,d)} d_{(i,j);l} - T^{(s,d)} g_z + r_z^{(s,d)} + T^{(s,d)} v_z\right)$$

- Or $\underset{\forall z}{Min} \sum_{\forall z \in p} P_{z,(s,d)}$

- Can be solved as a shortest path problem in a graph with edges represented by lightpaths and edge costs defined by term $P_{z,(s,d)}$

Column Generation (cont.)

SOLUTION:

- For any new lightpath z, the term $P_{z,(s,d)}$ can be reduced to:

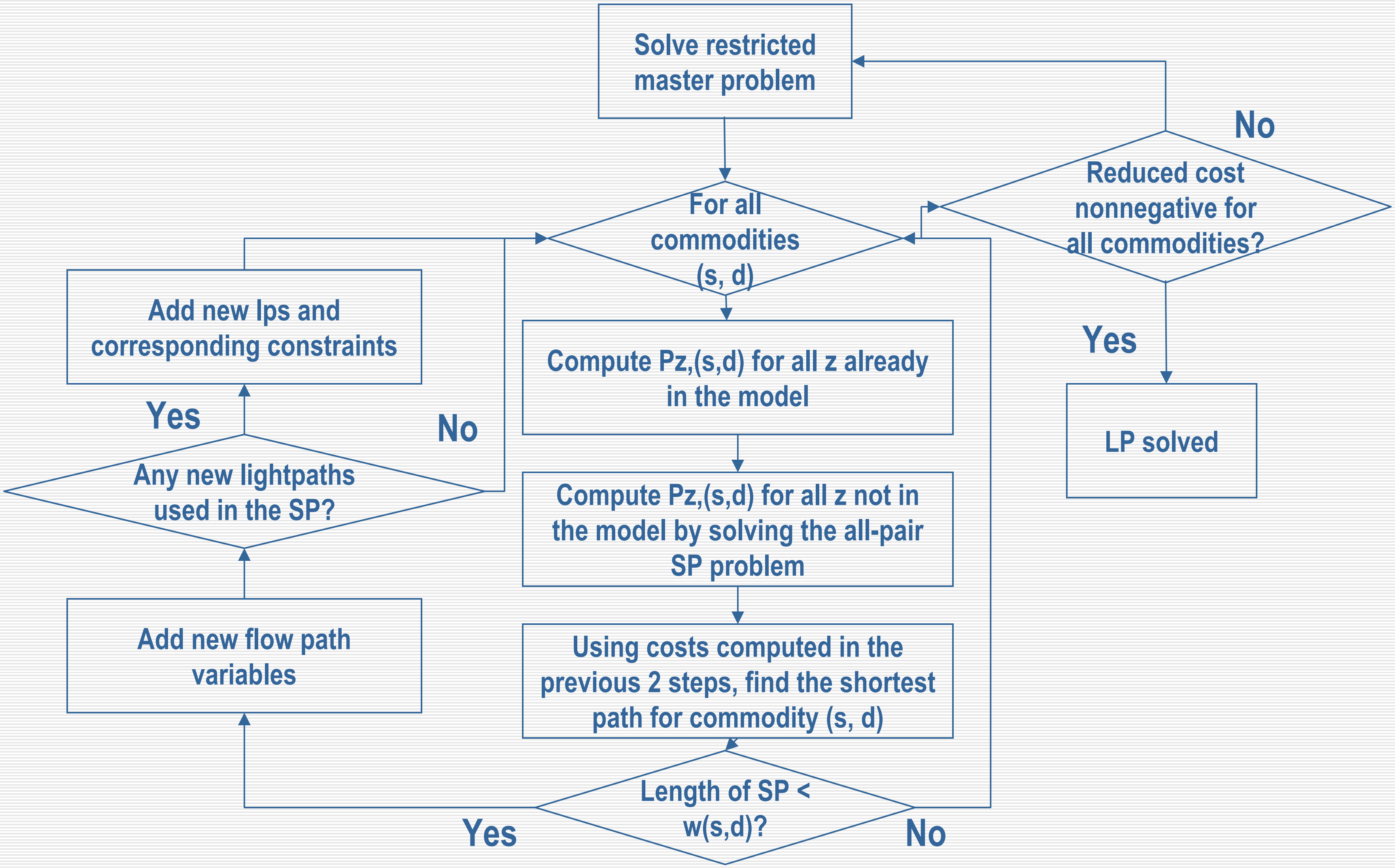
$$-T^{(s,d)}a_i - T^{(s,d)}b_j - \sum_{\forall (i,j);l \in z} T^{(s,d)} d_{(i,j);l}$$

- As we are looking for new lightpaths that will minimize the term $P_{z,(s,d)}$, we can solve the following for each pair of nodes:

$$Min \quad \{-a_i - b_j - \sum_{\forall (i,j);l \in z} d_{(i,j);l}\} \quad \text{or} \quad Min \quad \{- \sum_{\forall (i,j);l \in z} d_{(i,j);l}\}$$

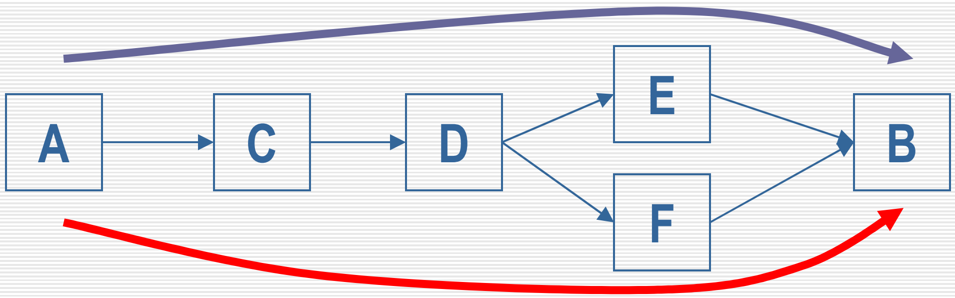
- Can be solved as an all-pair shortest path problem with edge costs defined by $-d_{(i,j);l}$

Column Generation – main steps



Branching Strategy

- Efficient branching strategy for ODIMCF problem (Barnhart et al.):
  - Identify 2 fractional paths for the fractional flow with greatest demand and create 2 children nodes using the following rule:



  - Let A be a set of arcs originating at divergence node (D). Define 2 subsets of arcs A1 and A2, such that $E \in A1$, $F \in A2$, $|A1| \approx |A2|$, $A1 \cap A2 = \emptyset$, and $A1 \cup A2 = A$.
  - Create one child node that does not use any arcs in set A1, and one child node that does not use any arcs in set A2
  - Important property: Proposed branching strategy does not destroy the structure of the pricing problem.

Branching Strategy (cont.)

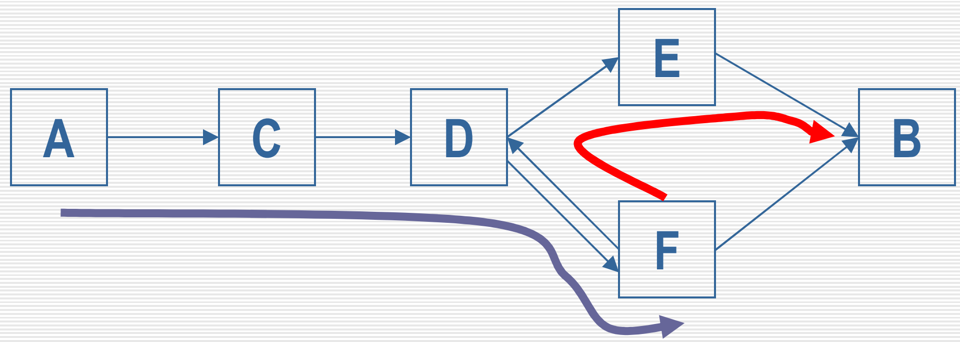
- Since a single flow path in the WDM OND problem may visit the same node more than once, we cannot apply similar branching strategy.

Example

Flow path A →B using lps:

A →F {A, C, D, F}

F →B {F, D, E, B}



- Solution: Apply branching strategy that prohibits use of certain arcs only for specific lightpaths of a given commodity

Branching Strategy (cont.)

- Step 1. Check if there are any commodities with fractional traffic. If there is no such commodity go to Step 4.
- Step 2. Identify commodity with greatest demand that has fractional lost traffic.
- Step 3. Create 2 new nodes:
  - Node 1: Set $H^{(s,d)} = 1$

    Do not serve demand for commodity (s,d) in the final solution
  - Node 2: Set $H^{(s,d)} = 0$

    Serve demand for commodity (s,d) in the final solution

Branching Strategy (cont.)

- Step 4. Identify 2 paths with the greatest fractions of flow for commodity (s, d) selected in Step 1.
- Step 5. If the 2 selected flow paths do not differ in the logical layer, go to Step 7.
- Step 6. Locate divergence node in the logical layer and create 2 new nodes (by first identifying 2 disjoint and exhaustive sets of lightpaths emanating from divergence node)
  - Node 1: for commodity (s, d) forbid all lps in the first set of arcs
  - Node 2: for commodity (s, d) forbid all lps in the second set of arcs

Branching Strategy (cont.)

- Step 7. Locate divergence node d in the physical layer, and identify wavelengths l1 and l2 on fibers originating at node d that are being used by flow paths identified in Step 2.
- Step 8. Identify origin and destination of the lp (say O'→D') corresponding to wavelenghts and fibers identified in Step 7.
- Step 9. Create 2 new nodes:
  - Node 1: If l1 and l2 are on different fibers do not allow allow commodity (s, d) to use any lps O' →D' that use fiber l2 belongs to. Otherwise, do not allow commodity (s, d) to use any lps O' →D' that use l2.
  - Node 2: If l1 and l2 are on different fibers do not allow allow commodity (s, d) to use any lps O' →D' that use fiber l1 belongs to. Otherwise, do not allow commodity (s, d) to use any lps O' →D' that use l1.

Applicability of the proposed BP algorithm to WDM OND with alternative design objectives

- Only minor modifications in computation of reduced cost are necessary when considering alternative design objectives, such as:
  - Quantity / cost of node equipment
  - Average hop distance over all flow paths in the network
- Overall Column Generation Algorithm and the Proposed Branching Strategy remain valid in all cases

Preliminary Computational Results

Node Nbr	Commodity Nbr	Demand	LB	UB	cpu (seconds)
5	20	H	0.11	0.73	0.516
5	20	L	0	0	0.188
5	10	H	0	0	0.109
5	10	L	0	0	0.094
7	42	H	4.018*	5.38	803.594
7	42	L	0	0.87	743.685
7	21	H	0.19	0.7	1.16
7	21	L	0	0.12	0.611
10	90	H	21.098*	23.18	4782.41
10	90	L	5.402*	8.22	4577.11
10	45	H	2.46*	4.1	3797.63
10	45	L	0	0.35	3617.45
20	380	H	152.451	155.44	2233.76
20	380	L	70.74	86.84	2587.49
20	190	H	52.794	57.29	2434.47
20	190	L	17.277	32.8	2280.26

Table 1. Minimizing lost traffic. Complete network with 2 fibers (fiber capacity: 2 lightpaths) between all pairs of nodes, 3 transmitters and 3 receivers at each node. Demand H: uniformly random [0.1, 1], L: uniformly random [0.1, 0.5].

Preliminary Computational Results

Node Nbr	Commodity Nbr	Demand	LB	UB	cpu (seconds)
5	20	H	23.825*	28	48.516
5	20	L	16.439*	20	29.781
5	10	H	15.002*	16	3.015
5	10	L	11.57*	14	6.891
7	42	H	51.521*	62	3796.53
7	42	L	35.340*	42	3678.02
7	21	H	26.457*	32	55.922
7	21	L	18.658*	22	59.797
10	90	H	112.596*	134	5646.34
10	90	L	75.637*	180**	3149.42
10	45	H	56.666*	68	4076.89
10	45	L	39.248*	50	3926.81

Table 2. Minimizing total number of transmitters and receivers in the network. Complete network with 2 fibers (fiber capacity: 2 lightpaths) between all pairs of nodes. Demand H: uniformly random [0.1, 1], L: uniformly random [0.1, 0.5].

Concluding Remarks

- Proposed Column Generation Algorithm for the WDM optical network design can be used to test optimality of solutions provided by existing heuristic procedures
- Application of the proposed procedures to WDM optical network design with alternative design objectives requires only minor modifications
- Efficiency of the proposed BP algorithm may be significantly improved by resolving degeneracy issue.

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